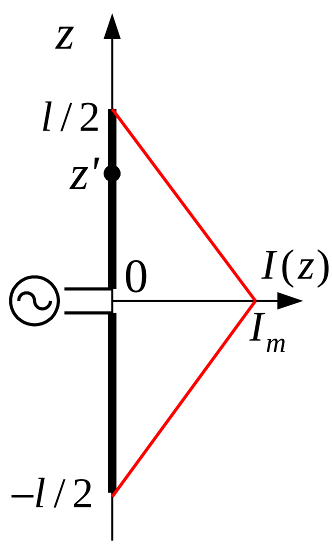
## Lecture 9: Linear Wire Antennas – Dipoles and Monopoles

(*Small electric dipole antenna. Finite-length dipoles. Half-wavelength dipole. Method of images - revision. Vertical infinitesimal dipole above a conducting plane. Monopoles. Horizontal infinitesimal dipole above a conducting plane.*)

The dipole and the monopole are arguably the two most widely used antennas across the UHF, VHF and lower-microwave bands. Arrays of dipoles are commonly used as base-station antennas in land-mobile systems. The monopole and its variations are perhaps the most common antennas for portable equipment, such as cellular telephones, cordless telephones, automobiles, trains, etc. It has attractive features such as simple construction, sufficiently broadband characteristics for voice communication, small dimensions at high frequencies. An alternative to the monopole antenna for hand-held units is the loop antenna, the microstrip patch antenna, the spiral antennas, inverted-L and inverted-F antennas, and others.

### 1. Small dipole




$$\frac{\lambda}{50} < l \le \frac{\lambda}{10} \tag{9.1}$$

If we assume that $R \approx r$ and condition (9.1) holds, the maximum phase error in $(\beta R)$ that can occur is

$$e_{\max} = \frac{\beta l}{2} = \frac{\pi}{10} \approx 18^\circ ,$$

at $\theta = 0^\circ$. *Reminder*: A maximum total phase error of $\pi / 8$ is acceptable since it does not affect substantially the integral solution for the vector potential **A**. The assumption $R \approx r$ is made here for both, the amplitude and the phase factors in the kernel of the VP integral.

The current is a triangular function of $z'$:

$$I(z') = \begin{cases} I_m \cdot \left(1 - \dfrac{z'}{l/2}\right), & 0 \le z' \le l/2 \\ I_m \cdot \left(1 + \dfrac{z'}{l/2}\right), & -l/2 \le z' \le 0 \end{cases} \tag{9.2}$$

The VP integral is obtained as

$$\mathbf{A} = \hat{\mathbf{z}}\frac{\mu}{4\pi}\left[\int_{-l/2}^{0} I_m\left(1+\frac{z'}{l/2}\right)\frac{e^{-j\beta R}}{R}dz' + \int_{0}^{l/2} I_m\left(1-\frac{z'}{l/2}\right)\frac{e^{-j\beta R}}{R}dz'\right]. \quad (9.3)$$

The solution of (9.3) is simple when we assume that $R \approx r$:

$$\mathbf{A} = \hat{\mathbf{z}}\frac{1}{2}\left[\frac{\mu}{4\pi} I_m l \frac{e^{-j\beta r}}{r}\right]. \quad (9.4)$$

The further away from the antenna the observation point is, the more accurate the expression in (9.4). Note that *the result in* (9.4) *is exactly one-half of the result obtained for* **A** *of an infinitesimal dipole of the same length*, if $I_m$ were the current uniformly distributed along the dipole. This is expected because we made the same approximation for $R$, as in the case of the infinitesimal dipole with a constant current distribution, and we integrated a triangular function along $l$, whose average is $I_0 = I_{av} = 0.5 I_m$.

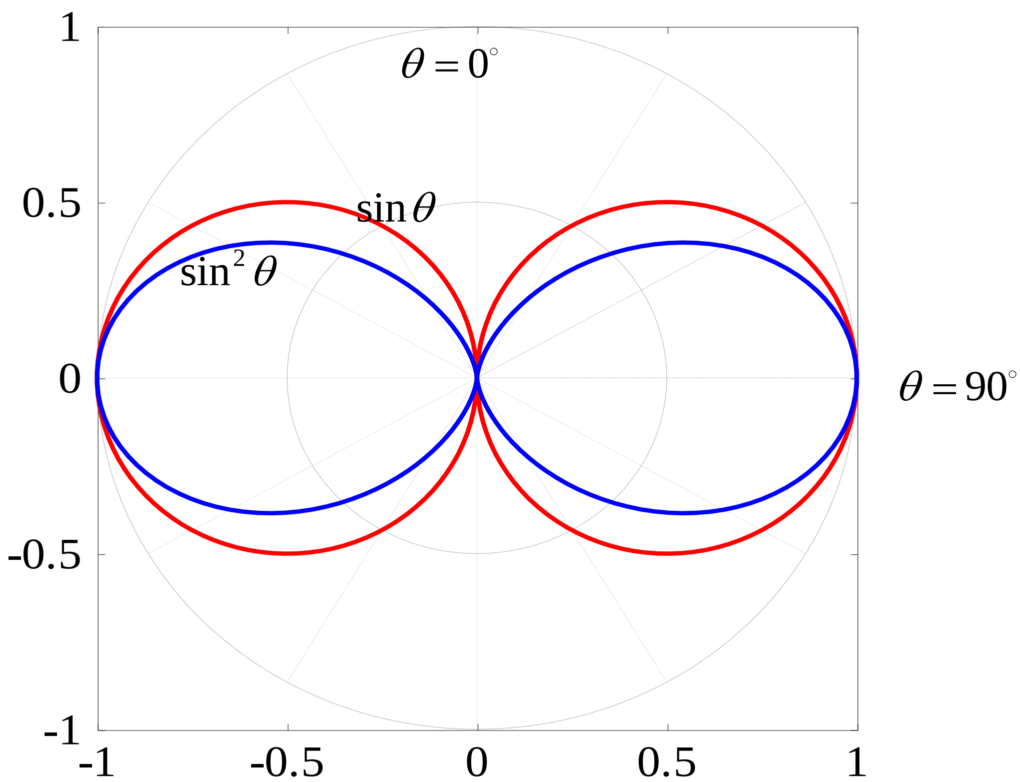
Therefore, we need not repeat all the calculations of the field components, power and antenna parameters. We make use of our knowledge of the infinitesimal dipole field. The far-field components of the small dipole are simply half those of the infinitesimal dipole:

$$\begin{aligned} E_\theta &\simeq j\eta \frac{\beta I_m l}{8\pi}\frac{e^{-j\beta r}}{r}\sin\theta \\ H_\varphi &\simeq j\frac{\beta I_m l}{8\pi}\frac{e^{-j\beta r}}{r}\sin\theta \;,\; \beta r \gg 1. \\ E_r &= E_\varphi = H_r = H_\theta = 0 \end{aligned} \quad (9.5)$$

The normalized field pattern is the same as that of the infinitesimal dipole:

$$\bar{E}(\theta,\varphi) = \sin\theta. \quad (9.6)$$

The power pattern: $$\bar{U}(\theta,\varphi)=\sin^2\theta \quad (9.7)$$




The beam solid angle:

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \cdot \sin\theta d\theta d\varphi,$$

$$\Omega_A = 2\pi \cdot \int_0^{\pi} \sin^3\theta d\theta = 2\pi \cdot \frac{4}{3} = \frac{8\pi}{3}$$

The directivity:

$$D_0 = \frac{4\pi}{\Omega_A} = \frac{3}{2} = 1.5. \quad (9.8)$$

As expected, the directivity, the beam solid angle as well as the effective aperture are the same as those of the infinitesimal dipole because the normalized patterns of both dipoles are the same.

The radiated power is four times less than that of an infinitesimal dipole of the same length and current $I_0 = I_m$ because the far fields are twice smaller in magnitude:

$$\Pi = \frac{1}{4} \cdot \frac{\pi}{3} \eta \left( \frac{I_m l}{\lambda} \right)^2 = \frac{\pi}{12} \eta \left( \frac{I_m l}{\lambda} \right)^2 . \tag{9.9}$$

As a result, the radiation resistance is also four times smaller than that of the infinitesimal dipole:

$$R_r = \frac{\pi}{6} \eta \left( \frac{l}{\lambda} \right)^2 = 20\pi^2 \left( \frac{l}{\lambda} \right)^2 . \tag{9.10}$$

## 2. Finite-length infinitesimally thin dipole

A good approximation of the current distribution along the dipole's length is the sinusoidal one:

$$I(z') = \begin{cases} I_0 \sin\left[ \beta \left( \frac{l}{2} - z' \right) \right], & 0 \le z' \le l/2 \\ I_0 \sin\left[ \beta \left( \frac{l}{2} + z' \right) \right], & -l/2 \le z' \le 0. \end{cases} \tag{9.11}$$

It can be shown that the VP integral

$$\mathbf{A} = \hat{\mathbf{z}} \frac{\mu}{4\pi} \int_{-l/2}^{l/2} I(z') \frac{e^{-j\beta R}}{R} dz' \tag{9.12}$$

has an analytical (closed form) solution. Here, however, we follow a standard approach used to calculate the far field for an arbitrary wire antenna. It is based on the solution for the field of the infinitesimal dipole. The finite-length dipole is subdivided into an infinite number of infinitesimal dipoles of length $dz'$. Each such dipole produces the elementary far field as

$$\begin{aligned} dE_\theta &\simeq j\eta\beta I_e(z') \frac{e^{-j\beta R}}{4\pi R} \sin\theta \cdot dz' \\ dH_\varphi &\simeq j\beta I_e(z') \frac{e^{-j\beta R}}{4\pi R} \sin\theta \cdot dz' \\ dE_r &= dE_\varphi = dH_r = dH_\theta = 0 \end{aligned} \tag{9.13}$$

where $R=[x^2+y^2+(z-z')^2]^{1/2}$ and $I_e(z')$ denotes the value of the current element at $z'$. Using the far-zone approximations,

$$\left| \begin{array}{l} \dfrac{1}{R} \simeq \dfrac{1}{r}, \text{ for the amplitude factor} \\ R \simeq r - z'\cos\theta, \text{ for the phase factor} \end{array} \right. \tag{9.14}$$

the following approximation of the elementary far field is obtained:

$$dE_\theta \simeq j\eta\beta I_e \frac{e^{-j\beta r}}{4\pi r} e^{j\beta z'\cos\theta} \cdot \sin\theta dz'. \tag{9.15}$$

Using the superposition principle, the total far field is obtained as

$$E_\theta = \int_{-l/2}^{l/2} dE_\theta \simeq j\eta\beta \frac{e^{-j\beta r}}{4\pi r} \cdot \sin\theta \cdot \int_{-l/2}^{l/2} I_e(z') e^{j\beta z'\cos\theta} dz'. \tag{9.16}$$

The *first factor*

$$g(\theta) = j\eta\beta \frac{e^{-j\beta r}}{r} \sin\theta \tag{9.17}$$

is called the ***element factor***. The element factor in this case is the far field produced by an infinitesimal dipole of unit current element $Il = 1 \ (\text{A} \times \text{m})$. The element factor is the same for any current element, provided the angle $\theta$ is always associated with the current axis. The *second factor*

$$f(\theta) = \int_{-l/2}^{l/2} I_e(z') e^{j\beta z'\cos\theta} dz' \tag{9.18}$$

is the ***space factor (or pattern factor, array factor)***. The pattern factor is dependent on the amplitude and phase distribution of the current at the antenna (the source distribution in space).

For the specific current distribution described by (9.11), the pattern factor is

$$f(\theta) = I_0 \left\{ \int_{-l/2}^{0} \sin\left[\beta\left(\frac{l}{2} + z'\right)\right] e^{j\beta z'\cos\theta} dz' + \int_{0}^{l/2} \sin\left[\beta\left(\frac{l}{2} - z'\right)\right] e^{j\beta z'\cos\theta} dz' \right\}. \tag{9.19}$$

The above integrals are solved having in mind that

$$\int \sin(a + b\cdot x) e^{c\cdot x} dx = \frac{e^{cx}}{b^2 + c^2}\left[c\sin(a+bx) - b\cos(a+bx)\right]. \tag{9.20}$$

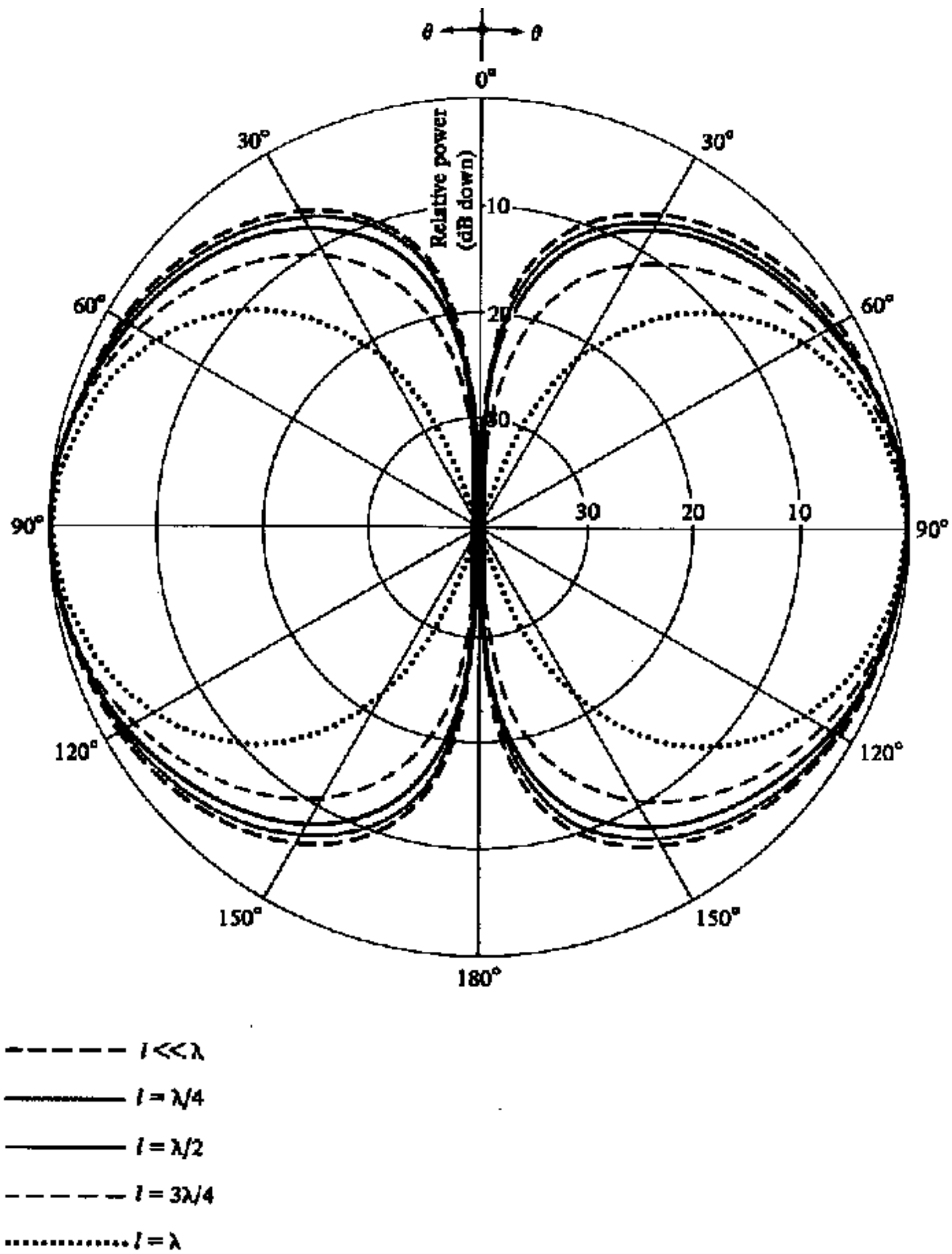
The far field of the finite-length dipole is obtained as

$$E_\theta = g(\theta)\cdot f(\theta) = j\eta I_0 \frac{e^{-j\beta r}}{2\pi r}\cdot\frac{\left[\cos\left(\frac{\beta l}{2}\cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)\right]}{\sin\theta}. \quad (9.21)$$

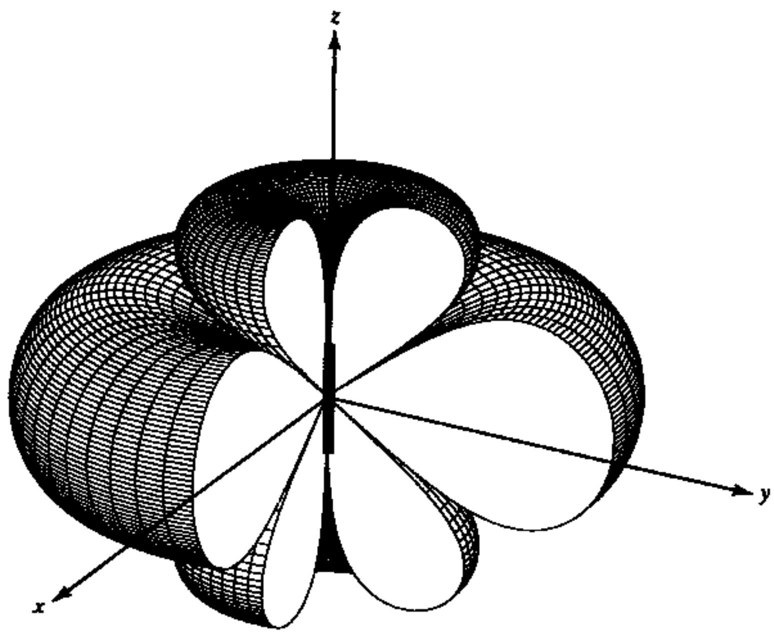
Amplitude pattern:

$$\bar{E}(\theta,\varphi) = \frac{\cos\left(\frac{\beta l}{2}\cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)}{\sin\theta}. \quad (9.22)$$

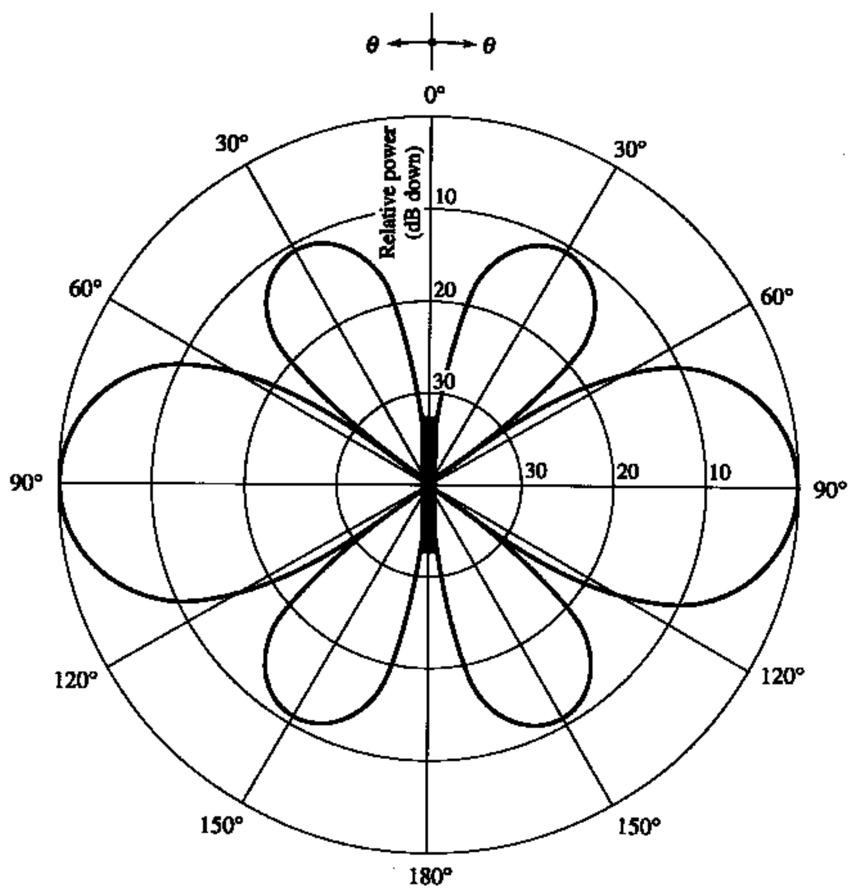
Patterns (in dB) for some dipole lengths $l \le \lambda$:



The 3-D pattern of the dipole $l = 1.25\lambda$:




(a) Three-dimensional




(b) Two-dimensional

Power pattern:

$$F(\theta,\varphi)=\frac{\left[\cos\left(\frac{\beta l}{2}\cos\theta\right)-\cos\left(\frac{\beta l}{2}\right)\right]^2}{\sin^2\theta}. \tag{9.23}$$

**Note:** The maximum of $F(\theta,\varphi)$ is not necessarily unity, but for $l<2\lambda$ the major maximum is always at $\theta=90^\circ$.

Radiated power

First, the far-zone power flux density is calculated as

$$\mathbf{P}=\hat{\mathbf{r}}\frac{1}{2\eta}|E_\theta|^2=\hat{\mathbf{r}}\eta\frac{I_0^2}{8\pi^2r^2}\left[\frac{\cos(0.5\beta l\cos\theta)-\cos(0.5\beta l)}{\sin\theta}\right]^2. \tag{9.24}$$

The total radiated power is given by the integral

$$\Pi=\oiint\mathbf{P}\cdot d\mathbf{s}=\int_0^{2\pi}\int_0^{\pi}P\cdot r^2\sin\theta d\theta d\varphi \tag{9.25}$$

$$\Pi=\eta\frac{I_0^2}{4\pi}\underbrace{\int_0^{\pi}\frac{\left[\cos(0.5\beta l\cos\theta)-\cos(0.5\beta l)\right]^2}{\sin\theta}d\theta}_{\Im}. \tag{9.26}$$

$\Im$ is solved in terms of the cosine and sine integrals:

$$\begin{aligned}\Im&=C+\ln(\beta l)-C_i(\beta l)+\frac{1}{2}\sin(\beta l)\left[S_i(2\beta l)-2S_i(\beta l)\right]+\\&+\frac{1}{2}\cos(\beta l)\left[C+\ln(\beta l/2)+C_i(2\beta l)-2C_i(\beta l)\right].\end{aligned} \tag{9.27}$$

Here,

$C\approx 0.5772$ is the Euler's constant,

$C_i(x)=\int_\infty^x\frac{\cos y}{y}dy=-\int_x^\infty\frac{\cos y}{y}dy$ is the cosine integral,

$S_i(x)=\int_0^x\frac{\sin y}{y}dy$ is the sine integral.

Thus, the radiated power can be written as

$$\Pi = \eta \frac{I_0^2}{4\pi} \cdot \Im. \tag{9.28}$$

Radiation resistance

The radiation resistance is defined as

$$R_r = \frac{2\Pi}{I_m^2} = \frac{I_0^2}{I_m^2} \cdot \frac{\eta}{2\pi} \cdot \Im \tag{9.29}$$

where $I_m$ is the maximum current magnitude along the dipole. If the dipole is half-wavelength long or longer ($l \geq \lambda/2$), $I_m = I_0$, see (9.11). However, if $l < \lambda/2$, then $I_m < I_0$ as per (9.11). If $l < \lambda/2$ holds, the maximum current is at the dipole center (the feed point $z' = 0$) and its value is

$$I_m = I_{(z'=0)} = I_0 \sin(\beta l/2) \tag{9.30}$$

where $\beta l/2 < \pi/2$, and, therefore $\sin(\beta l/2) < 1$. In summary,

$$\begin{aligned} I_m &= I_0 \sin(\beta l/2), && \text{if } l \leq \lambda/2 \\ I_m &= I_0, && \text{if } l > \lambda/2. \end{aligned} \tag{9.31}$$

Therefore,

$$\boxed{\begin{aligned} R_r &= \frac{\eta}{2\pi} \cdot \frac{\Im}{\sin^2(\beta l/2)}, && \text{if } l < \lambda/2 \\ R_r &= \frac{\eta}{2\pi} \cdot \Im && \text{, if } l \geq \lambda/2. \end{aligned}} \tag{9.32}$$

Directivity

The directivity is obtained as

$$D_0 = 4\pi \frac{U_{\max}}{\Pi} = 4\pi \frac{F_{\max}}{\int_0^{\pi} \int_0^{2\pi} F(\theta,\varphi) \sin\theta d\theta d\varphi} \tag{9.33}$$

where

$$F(\theta,\varphi) = \left[ \frac{\cos(0.5\beta l \cos\theta) - \cos(0.5\beta l)}{\sin\theta} \right]^2$$

is the power pattern [see (9.23)]. Finally,

$$D_0 = \frac{2F_{\max}}{\Im}. \tag{9.34}$$

Input resistance

The radiation resistance given in (9.32) is not necessarily equal to the input resistance because the current at the dipole center $I_{in}$ (if its center is the feed point) is not necessarily equal to $I_m$. In particular, $I_{in} \neq I_m$ if $l > \lambda/2$ and $l \neq (2n+1)\lambda/2$, $n$ is any integer. Note that when $l > \lambda/2$, $I_m = I_0$.

To obtain a general expression for the current magnitude $I_{in}$ at the center of the dipole (assumed also to be a feed point), we note that if the dipole is lossless, the input power is equal to the radiated power. Therefore,

$$P_{in} = \frac{|I_{in}|^2}{2} R_{in} = \Pi = \frac{|I_0|^2}{2} R_r \text{ for } l > \lambda/2. \tag{9.35}$$

Since the current at the center of the dipole ($z' = 0$) is [see (9.11)]

$$I_{in} = I_0 \sin(\beta l/2), \tag{9.36}$$

Then, according to (9.32),

$$R_{in} = \frac{R_r}{\sin^2(\beta l/2)} = \frac{\eta}{2\pi} \cdot \frac{\Im}{\sin^2(\beta l/2)}, \ l > \lambda/2. \tag{9.37}$$

For a short dipole ($l \leq \lambda/2$), $I_{in} = I_m$ and therefore

$$R_{in} = R_r = \frac{\eta}{2\pi} \cdot \frac{\Im}{\sin^2(\beta l/2)}, \ l \leq \lambda/2. \tag{9.38}$$

In summary, the dipole's input resistance, regardless of its length, depends on the integral $\Im$ as in (9.37)-(9.38), as long as the feed point is at the center.

Loss can be easily incorporated in the calculation of $R_{in}$ bearing in mind that the power-balance relation (9.35) can be modified as

$$P_{in} = \frac{|I_{in}|^2}{2} R_{in} = \Pi + P_{\text{loss}} = \frac{|I_m|^2}{2} R_r + P_{\text{loss}}. \tag{9.39}$$

In Lecture 4, p. 25, we have already obtained the expression for the loss of a dipole of length $l$:

$$P_{\text{loss}} = \frac{I_0^2 R_{hf}}{4}\left[1 - \frac{\sin(\beta l)}{\beta l}\right]. \tag{9.40}$$

## 3. Half-wavelength dipole

This is a classical and widely used thin wire antenna: $l = \lambda / 2$.

$$\boxed{\begin{aligned} E_\theta &= j\eta \frac{I_0 e^{-j\beta r}}{2\pi r} \cdot \frac{\cos(0.5\pi\cos\theta)}{\sin\theta} \\ H_\varphi &= E_\theta / \eta \end{aligned}} \tag{9.41}$$

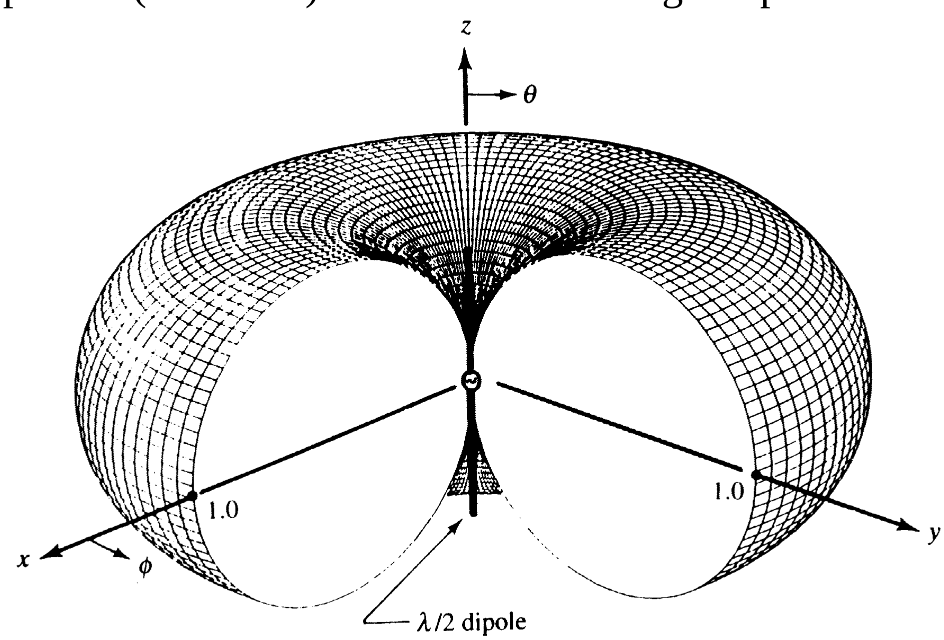
Radiated power flow density:

$$P = \frac{1}{2\eta} |E_\theta|^2 = \eta \frac{|I_0|^2}{8\pi^2 r^2} \underbrace{\left[\frac{\cos(0.5\pi\cos\theta)}{\sin\theta}\right]^2}_{F(\theta)\ -\ \text{normalized power pattern}} \approx \eta \frac{|I_0|^2}{8\pi^2 r^2} \sin^3\theta. \tag{9.42}$$

Radiation intensity:

$$U = r^2 P = \eta \frac{|I_0|^2}{8\pi^2} \underbrace{\left[\frac{\cos(0.5\pi\cos\theta)}{\sin\theta}\right]^2}_{F(\theta)\ -\ \text{normalized power pattern}} \approx \eta \frac{|I_0|^2}{8\pi^2} \sin^3\theta. \tag{9.43}$$

3-D power pattern (not in dB) of the half-wavelength dipole:



Radiated power

The radiated power of the half-wavelength dipole is a special case of the integral in (9.26):

$$\Pi = \eta \frac{|I_0|^2}{4\pi} \int_0^{\pi} \frac{\cos^2(0.5\pi\cos\theta)}{\sin\theta} d\theta \tag{9.44}$$

$$\Pi = \eta \frac{|I_0|^2}{8\pi} \int_0^{2\pi} \frac{1-\cos y}{y} dy \tag{9.45}$$

$$\mathcal{J} = 0.5772 + \ln(2\pi) - C_i(2\pi) \simeq 2.435 \tag{9.46}$$

$$\Rightarrow \underline{\Pi = 2.435 \frac{\eta}{8\pi} |I_0|^2 = 36.525 |I_0|^2} . \tag{9.47}$$

Radiation resistance:

$$R_r = \frac{2\Pi}{|I_0|^2} \approx 73 \;\; \Omega . \tag{9.48}$$

Directivity:

$$D_0 = 4\pi \frac{U_{\max}}{\Pi} = 4\pi \frac{U_{/\theta=90^\circ}}{\Pi} = \frac{4}{\mathfrak{I}} = \frac{4}{2.435} = 1.643 . \tag{9.49}$$

Maximum effective area:

$$A_e = \frac{\lambda^2}{4\pi} D_0 \simeq 0.13\lambda^2 . \tag{9.50}$$

Input resistance

Since $l = \lambda/2$,

$$R_{in} = R_r \approx 73 \;\; \Omega . \tag{9.51}$$

The imaginary part of the input impedance is approximately $+j42.5 \;\; \Omega$. To acquire maximum power transfer, this reactance has to be removed by matching (e.g., shortening) the dipole:

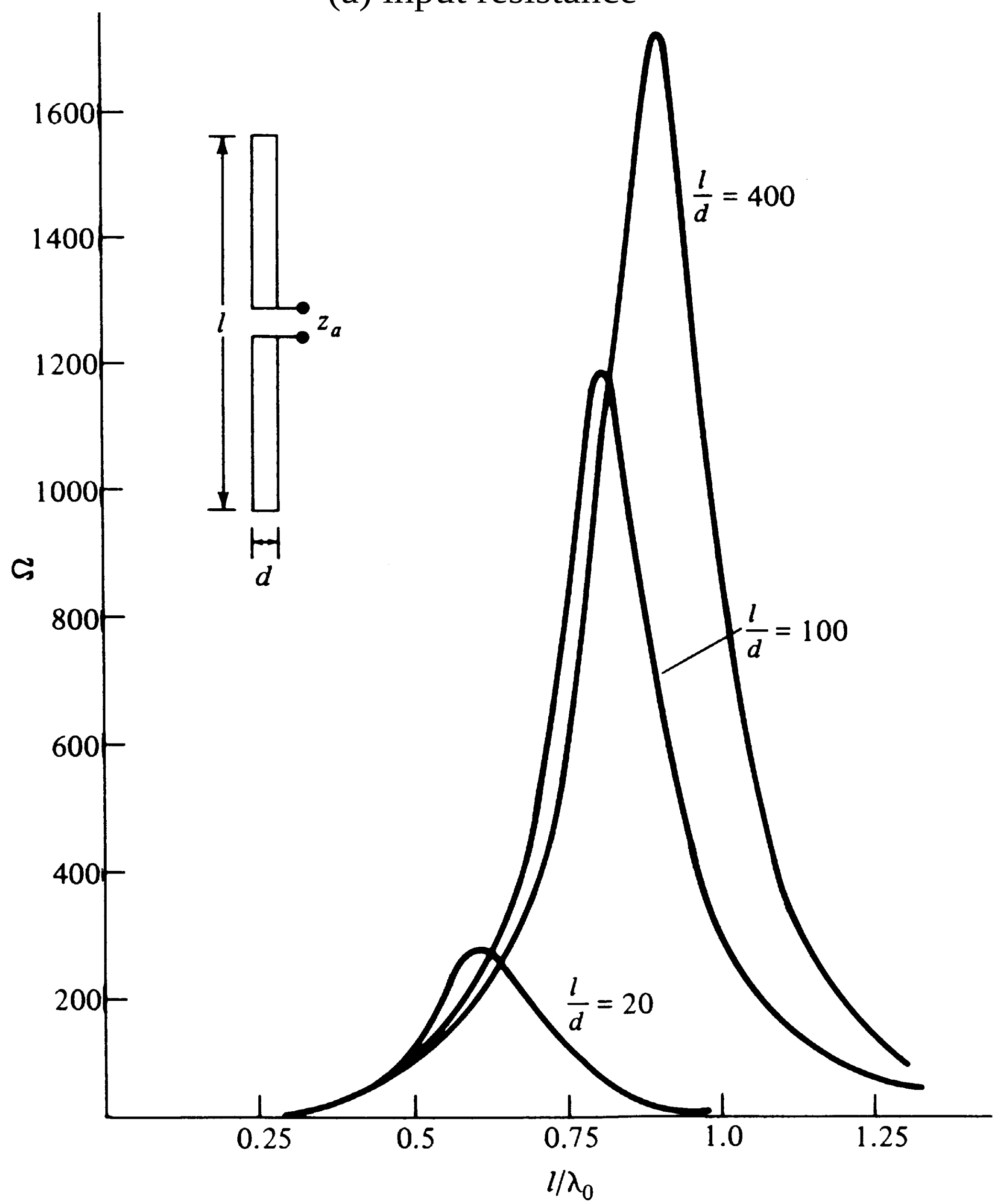
- thick dipole $l \approx 0.47\lambda$
- thin dipole $l \approx 0.48\lambda$.

The input impedance of the dipole is very frequency sensitive; i.e., it depends strongly on the ratio $l/\lambda$. This is to be expected from a resonant

narrow-band structure operating at or near resonance such as the half-wavelength dipole. We should also keep in mind that the input impedance is influenced by the capacitance associated with the physical junction to the transmission line. The structure used to support the antenna, if any, can also influence the input impedance. That is why the curves below describing the antenna impedance are only representative.

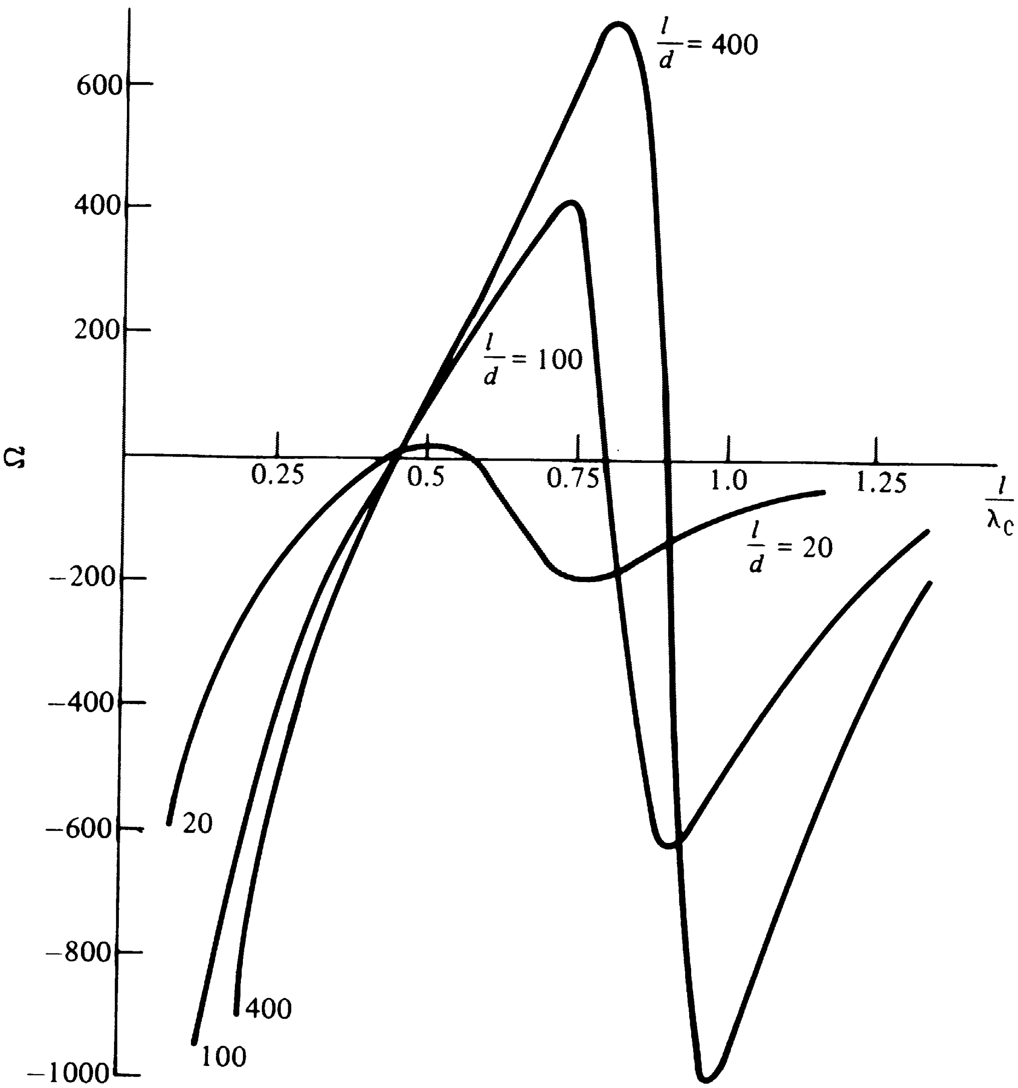
<u>Measurement results for the input impedance of a dipole</u>

(a) input resistance




Note the strong influence of the dipole diameter on its resonant properties.

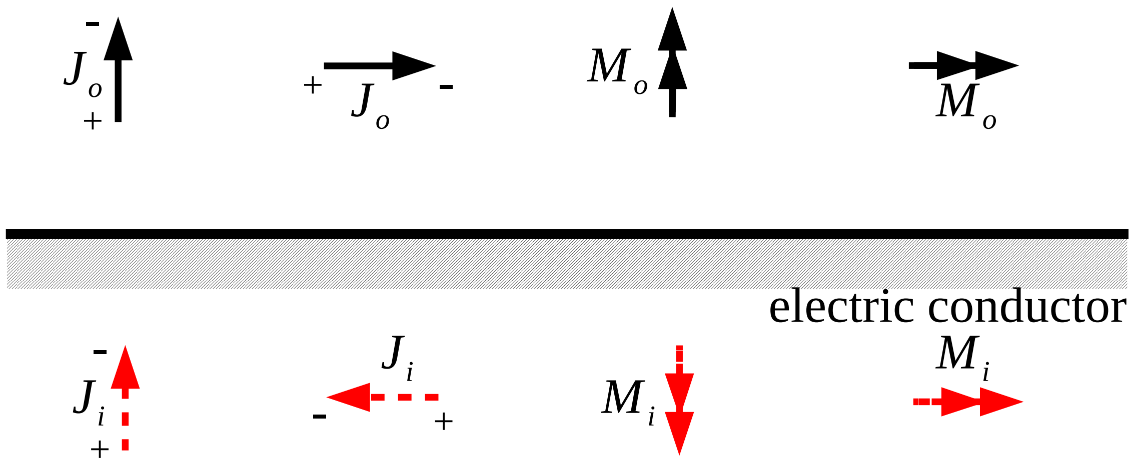
(b) input reactance

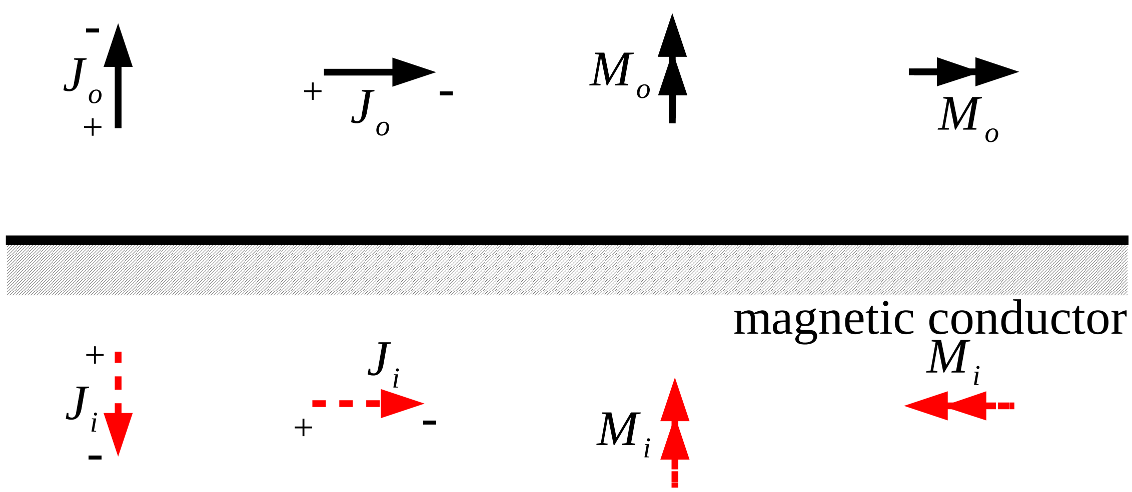



We can calculate the input resistance as a function of $l/\lambda$ using equations (9.29) and (9.37). These equations, however, are valid only for infinitesimally thin dipoles. Besides, it is important to compute the exact reactance, too. In practice, dipoles are most often tubular, and they have some finite diameter $d$. General-purpose numerical methods such as the ***method of moments*** (MoM) or the ***finite-difference time-domain*** (FDTD) method can be used to calculate the antenna impedance. When finite-thickness wire antennas are analyzed and no assumption is made for the current distribution along the wire, the MoM is applied to the classical Pocklington equation or to its variation, the Hallen equation. A classical method producing closed form

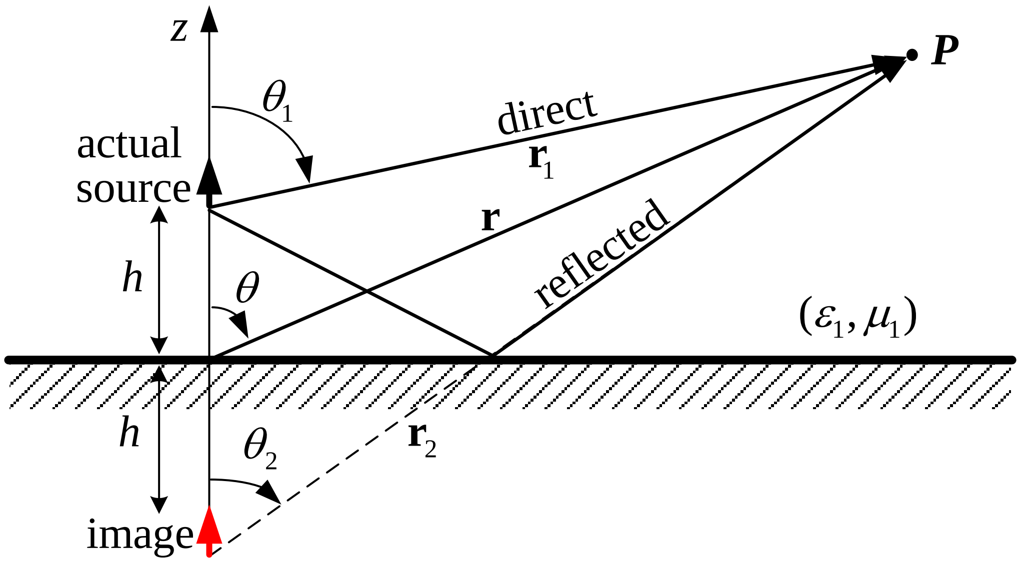
solutions for the self-impedance and the mutual impedance of straight-wire antennas is the ***induced electromotive force (emf) method***, which will be discussed later.

## 4. Method of images – revision



## 5. Vertical electric current element above perfect conductor




The field at the observation point $P$ is a superposition of the fields of the actual source and the image source, both radiating in a homogeneous medium of constitutive parameters $(\varepsilon_1, \mu_1)$. The actual source is a current element $(I_0 \Delta l)$ (infinitesimal dipole).

$$
\begin{aligned}
E_\theta^d &= j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r_1}}{4\pi r_1}\cdot\sin\theta_1,\\
E_\theta^r &= j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r_2}}{4\pi r_2}\cdot\sin\theta_2\ .
\end{aligned}
\tag{9.52}
$$

Expressing the distances $r_1 = |\mathbf{r}_1|$ and $r_2 = |\mathbf{r}_2|$ in terms of $r = |\mathbf{r}|$ and $h$ (using the cosine theorem) gives

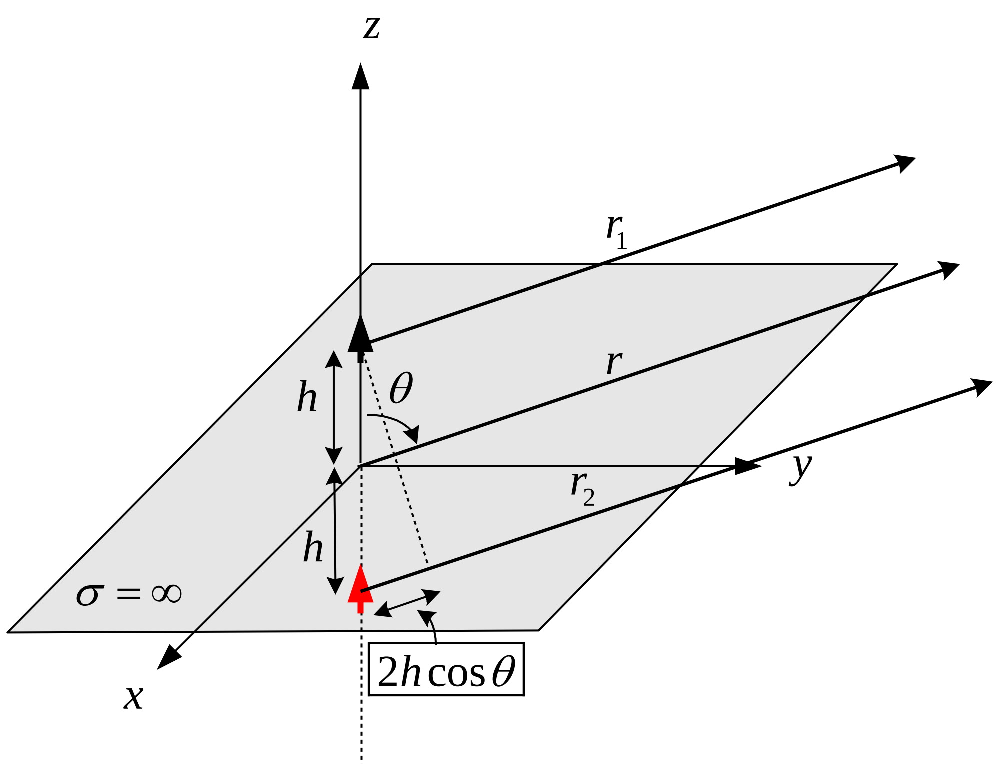
$$
\begin{aligned}
r_1 &= \sqrt{r^2 + h^2 - 2rh\cos\theta},\\
r_2 &= \sqrt{r^2 + h^2 - 2rh\cos(\pi-\theta)}\ .
\end{aligned}
\tag{9.53}
$$

We make use of the binomial expansions of $r_1$ and $r_2$ to approximate the amplitude and the phase terms, which simplify the evaluation of the total far field and the VP integral. For the amplitude term,

$$
\frac{1}{r_1} \simeq \frac{1}{r_2} \simeq \frac{1}{r}.
\tag{9.54}
$$

For the phase term, we use the second-order approximation (see also the geometrical interpretation below),

$$\begin{aligned} r_1 &\simeq r - h\cos\theta \\ r_2 &\simeq r + h\cos\theta \end{aligned} \tag{9.55}$$




The total far field is

$$E_\theta = E_\theta^d + E_\theta^r \tag{9.56}$$

$$E_\theta = j\eta\beta \frac{(I_0\Delta l)}{4\pi r} \cdot \sin\theta \left[ e^{-j\beta(r-h\cos\theta)} + e^{-j\beta(r+h\cos\theta)} \right] \tag{9.57}$$

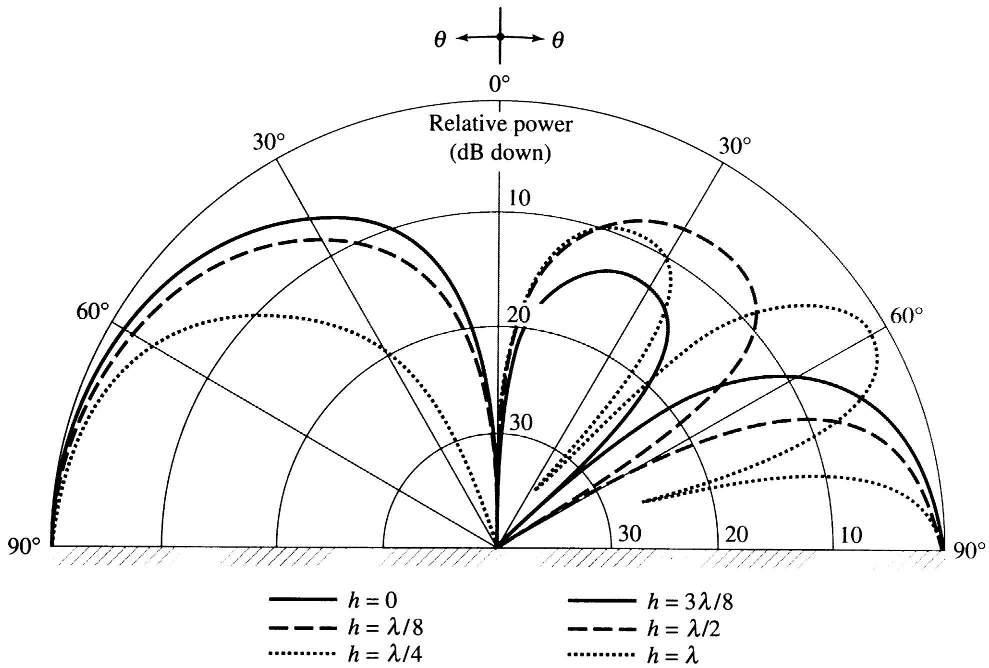
$$\boxed{\begin{aligned} &E_\theta = \underbrace{j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r}}{4\pi r}\sin\theta}_{g(\theta)} \cdot \underbrace{\left[2\cos(\beta h\cos\theta)\right]}_{f(\theta)}, \quad z \ge 0 \\ &E_\theta = 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad , \quad z < 0 \end{aligned}} \tag{9.58}$$

We note that the far-field expression can be again decomposed into two factors: the field of the elementary source $g(\theta)$ and the pattern factor (also array factor) $f(\theta)$.

The normalized power pattern is

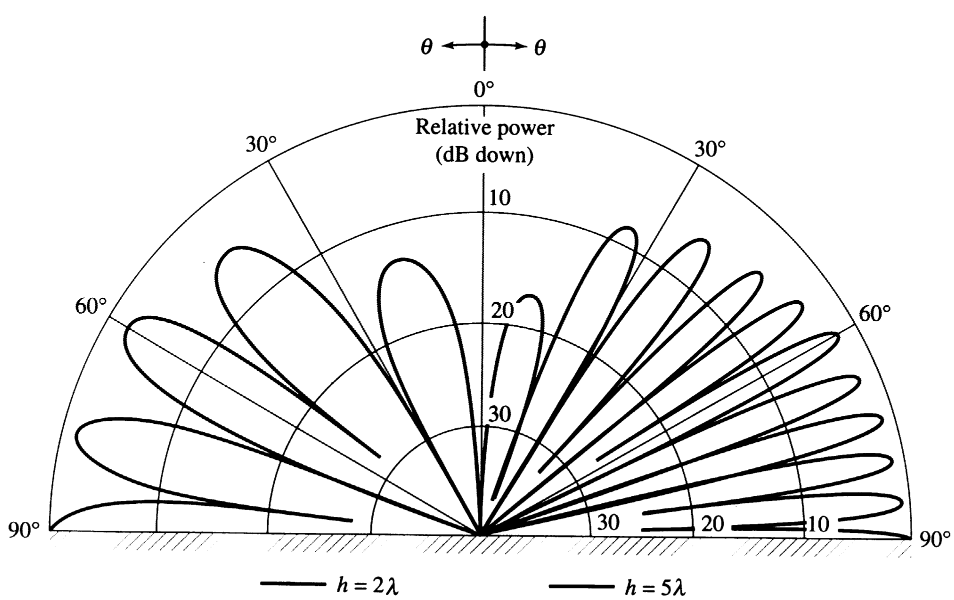
$$F(\theta)=\left[\sin\theta\cdot\cos\left(\beta h\cos\theta\right)\right]^2. \qquad (9.59)$$

Below, the elevation plane patterns are plotted for vertical infinitesimal electric dipoles of different heights above a perfectly conducting plane:




As the vertical dipole moves further away from the infinite conducting (ground) plane, more and more lobes are introduced in the power pattern. This effect is called ***scalloping*** of the pattern. The number of lobes is

$$n=\mathrm{nint}\left[(2h/\lambda)+1\right].$$



Total radiated power

$$\Pi = \oiint \mathbf{P}\cdot d\mathbf{s} = \frac{1}{2\eta}\int_0^{2\pi}\int_0^{\pi/2} |E_\theta|^2\, r^2 \sin\theta d\theta d\varphi,$$

$$\Pi = \frac{\pi}{\eta}\int_0^{\pi/2} |E_\theta|^2\, r^2 \sin\theta d\theta, \tag{9.60}$$

$$\Pi = \eta\beta^2 (I_0\Delta l)^2 \int_0^{\pi/2} \sin^2\theta\cdot\cos^2\left(\beta h\cos\theta\right) d\theta,$$

$$\Pi = \pi\eta\left(\frac{I_0\Delta l}{\lambda}\right)^2\left[\frac{1}{3}-\frac{\cos(2\beta h)}{(2\beta h)^2}+\frac{\sin(2\beta h)}{(2\beta h)^3}\right]. \tag{9.61}$$

- As $\beta h \to 0$, the radiated power of the vertical dipole above ground approaches twice the value of the radiated power of a dipole of the same length in free space.
- As $\beta h \to \infty$, the radiated power of both dipoles becomes the same.

Note:

$$\lim_{h\to 0}\left[-\frac{\cos\left(2\beta h\right)}{\left(2\beta h\right)^2}+\frac{\sin\left(2\beta h\right)}{\left(2\beta h\right)^3}\right]=\frac{1}{3}, \tag{9.62}$$

$$\lim_{h\to \infty}\left[-\frac{\cos\left(2\beta h\right)}{\left(2\beta h\right)^2}+\frac{\sin\left(2\beta h\right)}{\left(2\beta h\right)^3}\right]=0. \tag{9.63}$$

Radiation resistance

$$R_r = \frac{2\Pi}{|I_0|^2} = 2\pi\eta\left(\frac{\Delta l}{\lambda}\right)^2\left[\frac{1}{3}-\frac{\cos\left(2\beta h\right)}{\left(2\beta h\right)^2}+\frac{\sin\left(2\beta h\right)}{\left(2\beta h\right)^3}\right]. \tag{9.64}$$

- As $\beta h \to 0$, the radiation resistance of the vertical dipole above ground approaches twice the value of the radiation resistance of a dipole of the same length in free space:

$$R_{in}^{vdp} = 2R_{in}^{dp},\quad \beta h = 0. \tag{9.65}$$

- As $\beta h \to \infty$, the radiation resistance of both dipoles becomes the same.

Radiation intensity

$$U = r^2 P = r^2 \frac{|E_\theta|^2}{2\eta} = \frac{\eta}{2}\left(\frac{I_0 \Delta l}{\lambda}\right)^2 \sin^2\theta \cos^2\left(\beta h \cos\theta\right). \qquad (9.66)$$

The maximum of $U(\theta)$ occurs at $\theta = \pi / 2$:

$$U_{\max} = \frac{\eta}{2}\left(\frac{I_0 \Delta l}{\lambda}\right). \qquad (9.67)$$

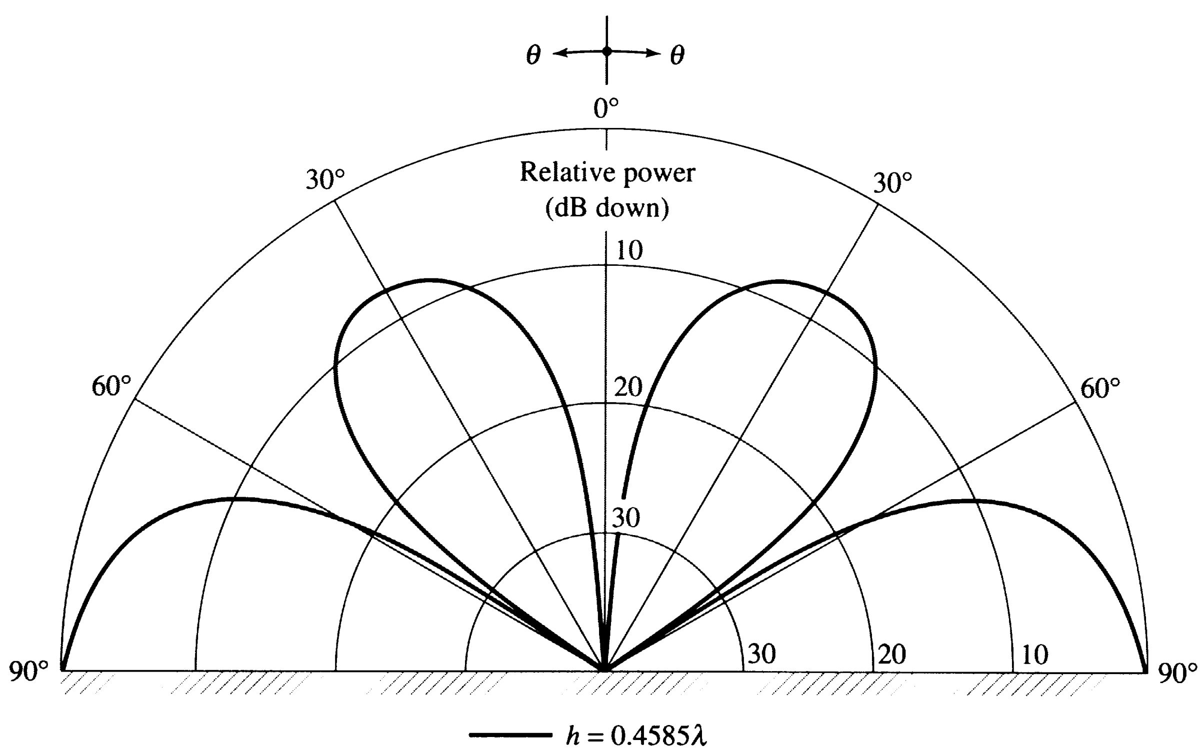
This value is 4 times greater than $U_{\max}$ of a free-space dipole of the same length.

Maximum directivity

$$D_0 = 4\pi \frac{U_{\max}}{\Pi} = \frac{2}{\frac{1}{3} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}}. \qquad (9.68)$$

If $\beta h = 0$, $D_0 = 3$, which is twice the maximum directivity of a free-space current element ($D_0^{id} = 1.5$).

The maximum of $D_0$ as a function of the height $h$ occurs when $\beta h =$ 2.881 ($h = 0.4585\lambda$). Then, $D_0 = 6.566_{/\beta h = 2.881}$.

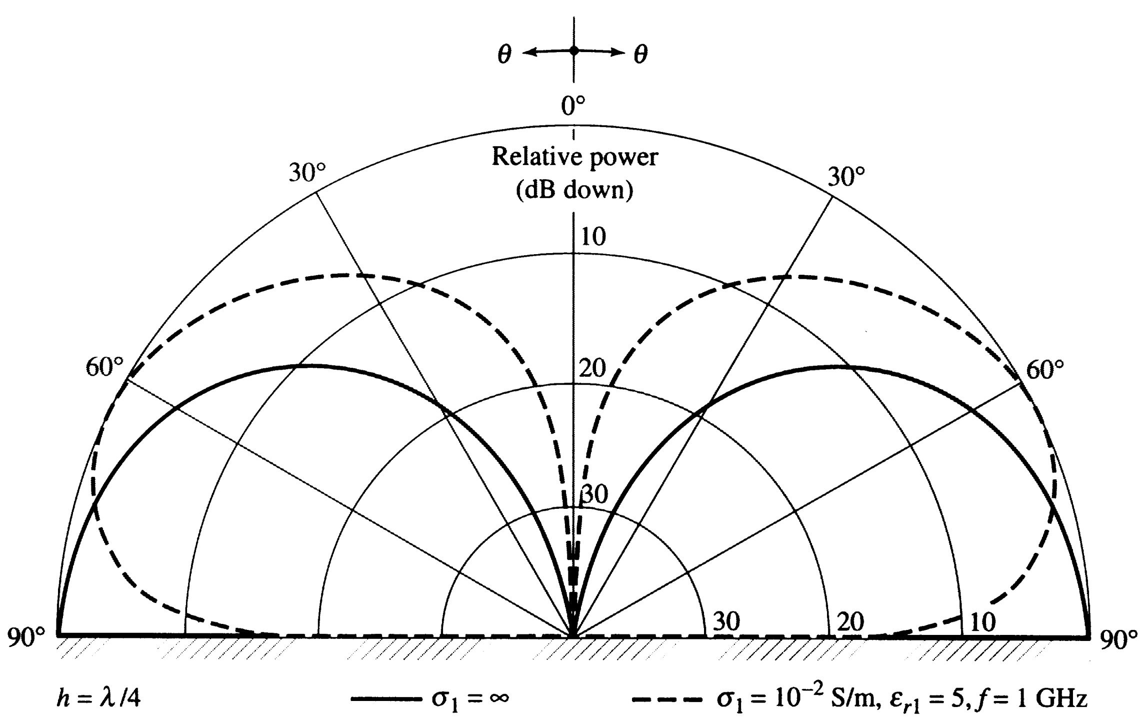


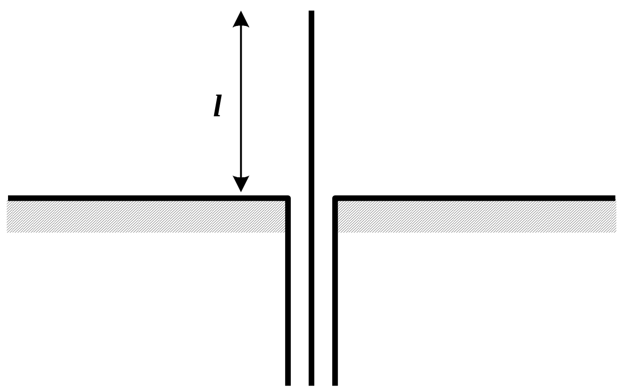
## 6. Monopoles

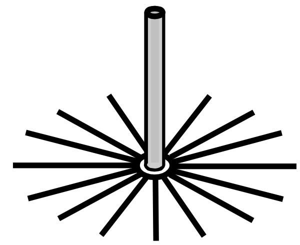
A monopole is a dipole that has been divided into half at its center where it is fed against a ground plane. It is normally $\lambda/4$ long (a ***quarter-wavelength monopole***), but it might by shorter when space restrictions dictate shorter lengths. Then, the monopole is a ***small monopole*** whose counterpart is the ***small dipole*** (see Section 1, this Lecture). Its current has linear distribution with its maximum at the feed point and its null at the end.

The vertical monopole is extensively used for AM broadcasting ($f$ = 500 to 1500 kHz, $\lambda$ = 200 to 600 m), because it is the shortest most efficient antenna at these frequencies as well as because vertically polarized waves suffer less attenuation at close-to-ground propagation. Vertical monopoles are widely used as base-station antennas in mobile communications, too.

Monopoles at base stations and radiobroadcast stations are supported by towers and guy wires. The guy wires must be separated into short enough ($\leq \lambda/8$) pieces insulated from each other to suppress any parasitic currents. Special care is taken when grounding the monopole. Usually, multiple radial wire rods, each $0.25-0.35\lambda$ long, are buried at the monopole base in the ground to simulate perfect ground plane, so that the pattern approximates closely the theoretical one, i.e., the pattern of the $\lambda/2$-dipole. Losses in the ground plane cause undesirable deformation of the pattern as shown below (infinitesimal dipole above an imperfect ground plane).





Monopole fed against a large solid ground plane

Practical monopole with radial wires to simulate perfect ground

Several important conclusions follow from the image theory and the discussion in Section 5:

- The field distribution in the upper half-space is the same as that of the respective free-space dipole.
- The currents and charges on a monopole are the same as on the upper half of its dipole counterpart but the terminal voltage is only half that of the dipole. The input impedance of a monopole is therefore only half that of the respective dipole:

$$Z_{in}^{mp} = \frac{1}{2} Z_{in}^{dp}. \tag{9.69}$$

- The total radiated power of a monopole is half the power radiated by its dipole counterpart since it radiates in half-space (but its field is the same). As a result, the beam solid angle of the monopole is half that of the respective dipole and its directivity is twice the directivity of the dipole:

$$D_0^{mp} = \frac{4\pi}{\Omega_A^{mp}} = \frac{4\pi}{\frac{1}{2}\Omega_A^{dp}} = 2D_0^{dp}. \tag{9.70}$$

## The quarter-wavelength monopole

This is a straight wire of length $l = \lambda / 4$ mounted over a ground plane. From the discussion above, it can be expected that the quarter-wavelength monopole is very similar to the half-wavelength dipole (in the hemisphere above the ground plane).

- Its radiation pattern is the same as that of a free-space $\lambda / 2$-dipole, but it is non-zero only for $0^\circ < \theta \le 90^\circ$ (above ground).
- The field expressions are the same as those of the $\lambda / 2$-dipole.
- The radiated power of the $\lambda / 4$-monopole is half that of the $\lambda / 2$-dipole.
- The radiation resistance of the $\lambda / 4$-monopole is half that of the $\lambda / 2$-dipole: $Z_{in}^{mp} = 0.5 Z_{in}^{dp} = 0.5(73 + j42.5) = 36.5 + j21.25, \ \ \Omega$.
- The directivity of the $\lambda / 4$-monopole is
$$D_0^{mp} = 2 D_0^{dp} = 2 \cdot 1.643 = 3.286.$$

## Some approximate formulas for rapid calculations of the input resistance of a dipole and the respective monopole:

Let
$$\left| \begin{array}{l} G = \dfrac{\beta l}{2} = \pi \dfrac{l}{\lambda}, \ \text{ for } \ \text{dipole} \\ G = \beta l = 2\pi \dfrac{l}{\lambda}, \ \text{ for } \ \text{monopole} \end{array} \right.$$

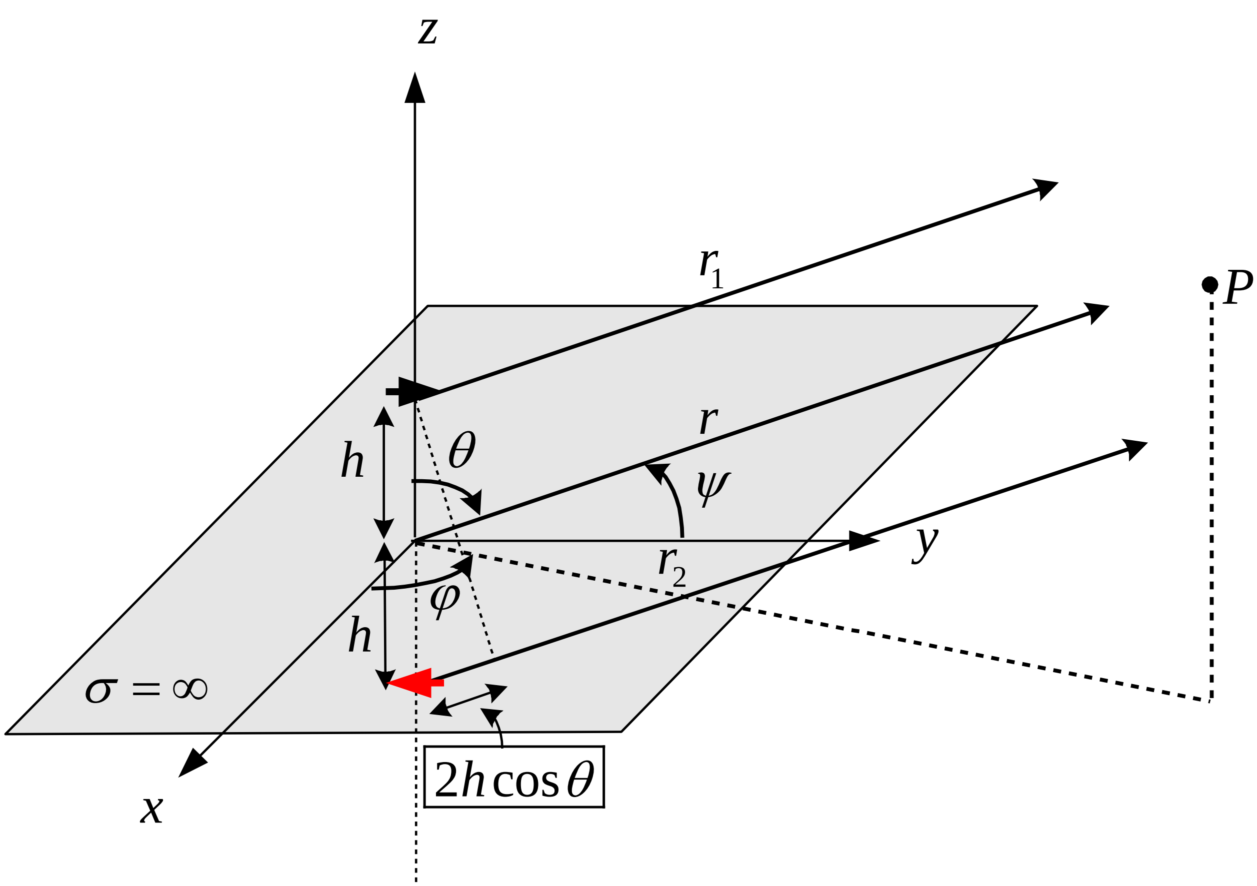
Then,

- if $0 < G < \dfrac{\pi}{4}$
$$\left| \begin{array}{ll} R_{in} = 20 G^2 & \text{, dipole} \\ R_{in} = 10 G^2 & \text{, monopole} \end{array} \right.$$
- if $\dfrac{\pi}{4} < G < \dfrac{\pi}{2}$
$$\left| \begin{array}{ll} R_{in} = 24.7 G^{2.5} & \text{, dipole} \\ R_{in} = 12.35 G^{2.5} & \text{, monopole} \end{array} \right.$$
- if $\dfrac{\pi}{2} < G < 2$

$$\begin{cases} R_{in} = 11.14G^{4.17} & \text{,dipole} \\ R_{in} = 5.57G^{4.17} & \text{,monopole} \end{cases}$$

## 7. Horizontal current element above a perfectly conducting plane

The analysis is analogous to that of a vertical current element above a ground plane. The difference arises in the element factor $g(\theta)$ because of the horizontal orientation of the current element. Let us assume that the current element is oriented along the $y$-axis, and the angle between $\vec{r}$ and the dipole's axis ($y$-axis) is $\psi$.




$$\mathbf{E}(P) = \mathbf{E}^d(P) + \mathbf{E}^r(P), \tag{9.71}$$

$$E_\psi^d = j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r_1}}{4\pi r_1}\sin\psi, \tag{9.72}$$

$$E_\psi^r = -j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r_2}}{4\pi r_2}\sin\psi. \tag{9.73}$$

We can express the angle $\psi$ in terms of $(\theta,\varphi)$:

$$\cos\psi = \hat{\mathbf{y}}\cdot\hat{\mathbf{r}} = \hat{\mathbf{y}}\cdot(\hat{\mathbf{x}}\sin\theta\cos\varphi + \hat{\mathbf{y}}\sin\theta\sin\varphi + \hat{\mathbf{z}}\cos\theta)$$
$$\Rightarrow \cos\psi = \sin\theta\sin\varphi$$
$$\Rightarrow \sin\psi = \sqrt{1-\sin^2\theta\sin^2\varphi} \tag{9.74}$$

The far-field approximations are:

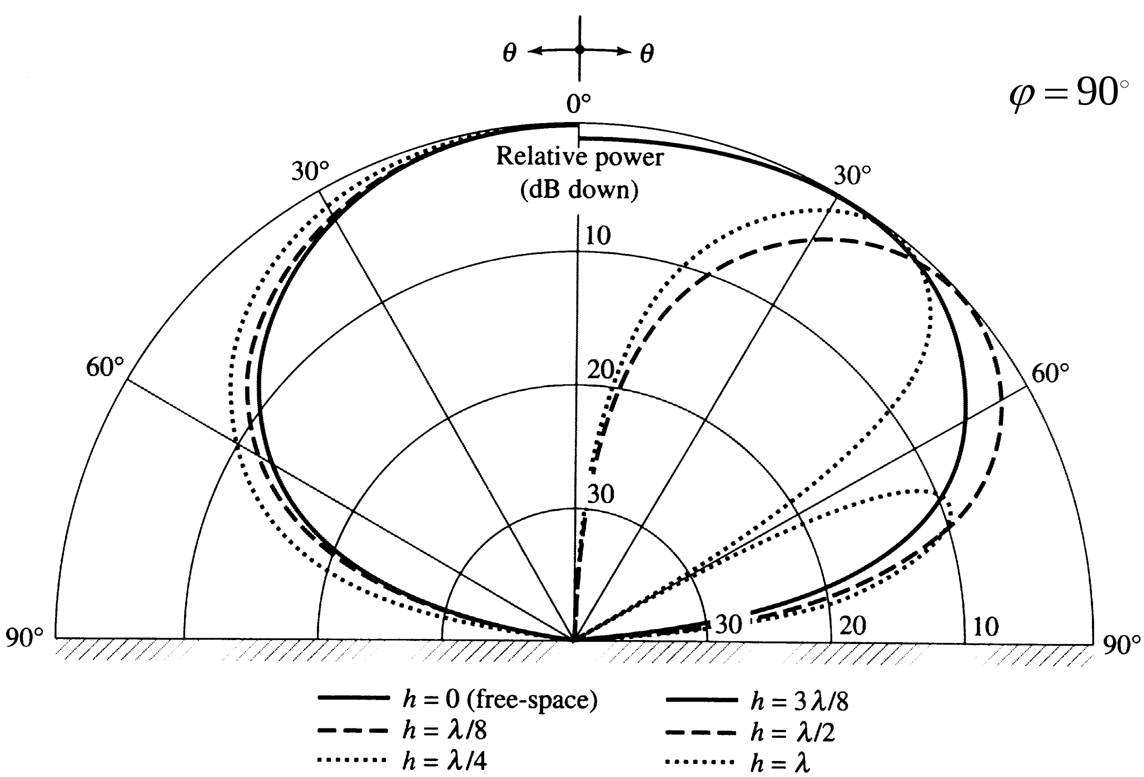
$$\left| \begin{array}{l} \dfrac{1}{r_1} = \dfrac{1}{r_2} = \dfrac{1}{r}, \ \text{for the amplitude term} \\ \left.\begin{array}{l} r_1 \simeq r - h\cos\theta \\ r_2 \simeq r + h\cos\theta \end{array}\right\} \ \text{for the phase term.} \end{array} \right.$$

The substitution of the far-field approximations and equations (9.72), (9.73), (9.74) into the total field expression (9.71) yields

$$E_\psi(\theta,\varphi) = \underbrace{j\eta\beta(I_0\Delta l)\frac{e^{-j\beta r}}{4\pi r}\sqrt{1-\sin^2\theta\sin^2\varphi}}_{\text{element factor } g(\theta,\varphi)}\underbrace{\left[2j\sin\left(\beta h\cos\theta\right)\right]}_{\text{array factor } f(\theta,\varphi)}. \tag{9.75}$$

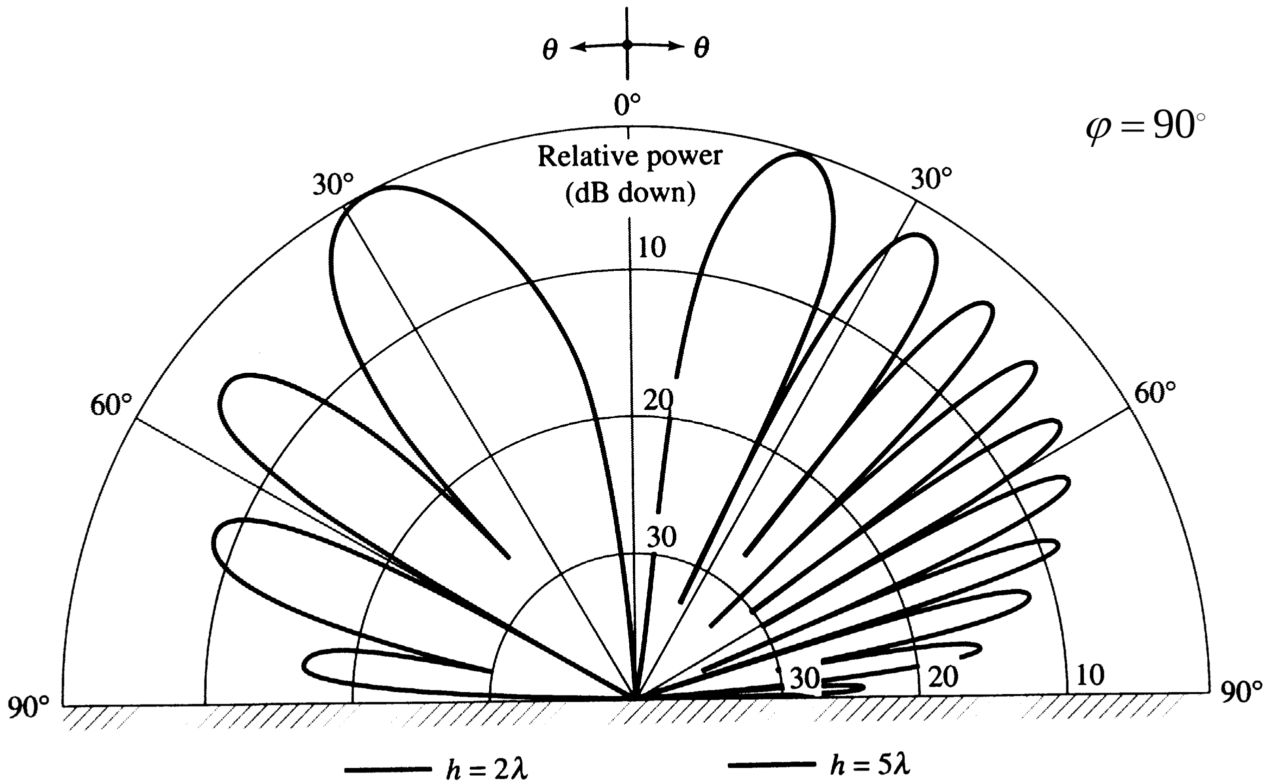
<u>The normalized pattern</u>

$$F(\theta,\varphi) = \left(1-\sin^2\theta\sin^2\varphi\right)\cdot\sin^2\left(\beta h\cos\theta\right) \tag{9.76}$$



As the height increases beyond a wavelength ($h > \lambda$), scalloping appears with the number of lobes being

$$n = \text{nint}\left(2\frac{h}{\lambda}\right). \quad (9.77)$$




Following a procedure similar to that of the vertical dipole, the radiated power and the radiation resistance of the horizontal dipole can be found:

$$\Pi = \frac{\pi}{2}\eta\left(\frac{I_0\Delta l}{\lambda}\right)^2 \underbrace{\left[\frac{2}{3} - \frac{\sin(2\beta h)}{2\beta h} - \frac{\cos(2\beta h)}{(2\beta h)^2} + \frac{\sin(2\beta h)}{(2\beta h)^3}\right]}_{R(\beta h)} \quad (9.78)$$

$$R_r = \pi\eta\left(\frac{\Delta l}{\lambda}\right)^2 \cdot R(\beta h). \quad (9.79)$$

By expanding the sine and the cosine functions into series, it can be shown that for small values of $(\beta h)$ the following approximation holds:

$$R_{/\beta h \to 0} \approx \frac{32\pi^2}{15}\left(\frac{h}{\lambda}\right)^2. \quad (9.80)$$

It is also obvious that if $h=0$, then $R_r=0$ and $\Pi=0$. This is to be expected because the dipole is short-circuited by the ground plane.

Radiation intensity

$$U=\frac{r^2}{2\eta}|\mathbf{E}_\psi|^2=\frac{\eta}{2}\left(\frac{I_0\Delta l}{\lambda}\right)^2\left(1-\sin^2\theta\sin^2\varphi\right)\sin^2\left(\beta h\cos\theta\right) \qquad (9.81)$$

The maximum value of (9.81) depends on whether $(\beta h)$ is less than $\pi/2$ or greater:

- If $\beta h\le\frac{\pi}{2}$ $\left(h\le\frac{\lambda}{4}\right)$

$$U_{\max}=\frac{\eta}{2}\left(\frac{I_0\Delta l}{\lambda}\right)^2\sin^2\left(\beta h\right)_{/\theta=0^\circ}. \qquad (9.82)$$

- If $\beta h>\frac{\pi}{2}$ $\left(h>\frac{\lambda}{4}\right)$

$$U_{\max}=\frac{\eta}{2}\left(\frac{I_0\Delta l}{\lambda}\right)^2_{/\theta=\arccos\left(\frac{\pi}{2\beta h}\right),\varphi=0^\circ}. \qquad (9.83)$$

Maximum directivity

For small $\beta h$, $D_0=7.5\left(\frac{\sin(\beta h)}{\beta h}\right)^2$.